A method to compute the pressure field of geophysical flows

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The improvement of Doppler radars technology provides a reliable estimation of the velocity wind field $\mathbf{v}$. The Doppler radars, however, cannot estimate thermodynamic variables such as the pressure and temperature, which are essential for several meteorological studies [1]. On the other hand, the complete specification of all relevant variables at a given instant, are necessary to specify the initial conditions of numerical simulations with prediction models [2]. The purpose of this work is to propose an scheme to compute the pressure p at a given instant when the velocity field $\mathbf{v}$ is known.

The main scheme to compute p from $\mathbf{v}$ is based on the momentum equation $-\rho^{-1}\nabla p = \mathbf{E}$, where we set $\mathbf{E} \equiv \mathbf{a} + 2(\mathbf{\Omega} \times \mathbf{v}) + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R}) - \mathbf{g} - \mathbf{F}$, which leads to the so-called Poisson pressure equation (PPE) $-\nabla^2 p = \nabla \cdot \rho \mathbf{E}$ which is solved with the Neumann boundary condition $\mathbf{n} \cdot \nabla p = \mathbf{n} \cdot \mathbf{E}$ on the boundary of the bounded region of interest $\mathbb{R}$ [3]. This scheme has some practical difficulties when we are considering geophysical flows since: (i) the velocity $\mathbf{v}$ is not known exactly and the numerical differentiation involved in $\mathbf{E}$ magnifies the errors of estimations of $\mathbf{v}$, (ii) the density ρ is not known exactly, particularly in the meteorological case. The scheme proposed in this work solves part of these problems. The scheme, which was used in previous works [4] to circumvent precisely the solution of the PPE in some particular problems, consists in solving the ordinary differential equation

$$\frac{d\ f(\xi_s, t, \theta)}{d\xi_s} = -A(\mathbf{R}, \mathbf{v}, t)\cos\theta - B(\mathbf{R}, \mathbf{v}, t)\sin\theta \quad \text{subject to } f(\xi_s = 0, t, \theta) = z_0\ , \tag{1}$$

where the coefficients A and B are obtained from the momentum equation, $f(\xi_s, t, \theta)$ is the equation of the isobar defined by the intersection of the constant pressure surface $S(x, y, z, z_0)$ that cross the z axis at $z = z_0$ and a plane parameterized with ξ_s, θ. The Eq. (1) is notable since it does not contain the density ρ. In this work the following results are proved. If the vector $\mathbf{E}$ is known in a region $\mathbb{R}$, then: (i) the sole solution of Eq. (1) permits the calculation of constant pressure surfaces $S(x, y, z, z_0)$ in $\mathbb{R}$ without any additional thermodynamic data. (ii) If, additionally, we known one radiosonde in the region $\mathbb{R}$, then we can compute the pressure $p(x, y, z)$ at each point (x, y, z) in $\mathbb{R}$, without any density and temperature data. Some examples show that the solution of Eq. (1) provide analytic expressions of the pressure $p(x, y, z)$ in cases where the PPE requires the density field ρ and has to be solved numerically. In the case of an inviscid flow a Bernoulli equation can be obtained only if additional thermodynamic information is used. For instance, if we suppose that the flow is barotropic, the we can obtain a Bernoulli equation. In this case it is shown analytically that the information obtained by solving (1) coincides with that from the Bernoulli equation. Operational and theoretical applications of the scheme are outlined and scale analysis is used to estimate the spatial validity region of the method.

References.

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