# Approximate solution of bidimensional momentum equations for inviscid atmospheric flows

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It is generally accepted [1] that the movement of large scale atmospheric flows is governed by the bidimensional Euler equations

$$\begin{aligned} \frac{du}{dt} - \frac{uv\tan\phi}{r_e} - 2\Omega v \sin\phi &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{dv}{dt} + \frac{u^2\tan\phi}{r_e} + 2\Omega u \sin\phi &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \end{aligned} \tag{1}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$. In this work we give an approximate analytic solution $u_a$, $v_a$ of Eqs. (1), which was obtained by a perturbation method [2].

The accuracy of $u_a$, $v_a$ is estimated by computing the solutions $u$, $v$ corresponding to the inertial trajectories of an atmospheric particle, since in this case $u$, $v$ can be computed analytically [3]. There are several regimens of qualitatively different inertial trajectories, regimens that can be classified with the aid of the Hamiltonian formulation of (1) [3]. The results reported in this work show that $u_a$, $v_a$ are close to $u$, $v$ as $t$ goes from 0 to $t_{\max}$ ~18 hrs independently of the regimen of movement. The criterion to estimate $t_{\max}$ is the following: if $\mathbf{r}_a(t)$ and $\mathbf{r}(t)$ are the inertial trajectories obtained from $\{u_a, v_a\}$ and $\{u, v\}$, respectively, we set $d(\mathbf{r}_a, \mathbf{r}) = \|\mathbf{r}_a(t) - \mathbf{r}(t)\|^{1/2}$ and define $t_{\max} = \inf\{t : d(\mathbf{r}_a, \mathbf{r}) = 10 \text{ km}\}$. The reliability time interval $[0, t_{\max}]$ increases as the latitude of the movement also does, in such a way that for latitudes from $55^\circ$ to $85^\circ$ the approximations $u_a$, $v_a$ are close to $u$, $v$ for $t \in [0, t_{\max} \sim 35 \text{ hrs}]$. Additional graphs of $t$ v.s. $\{u_a, v_a\}$, $\{u, v\}$ confirm the closness between $u_a, v_a$ and $u, v$. These results suggest that the approximations $u_a$, $v_a$ can be used for operational and theoretical purposes.

References.

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