A study of the asymptotic character of the geostrophic wind

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The dynamics of large scale atmospheric flows at midlatitudes is governed by the momentum equations

$$\begin{aligned} \frac{du}{dt} - \frac{uv\tan\phi}{r_e} - 2\Omega v\sin\phi &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{dv}{dt} + \frac{u^2\tan\phi}{r_e} + 2\Omega u\sin\phi &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \end{aligned} \tag{1}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$, together with the energy and mass conservation equations [1]. An approximate solution of Eqs. (1) is obtained by a scale analysis which leads to the well-known geostrophic wind $\mathbf{v}_g = u_g\mathbf{i} + v_g\mathbf{j} = \mathbf{k}\times\rho^{-1}f^{-1}\nabla p$ $(f = 2\Omega\sin\phi)$. Several authors have studied the asymptotic character of $\mathbf{v}_g$ in order to obtain, e.g., the proper initial conditions that should be used when $\mathbf{v}_g$ is used into (1) for short-range synoptic forecasts [2]. In fact, as is know [2,3], asymptotic solutions of (2) are valid on a time interval $[\delta, t]$ with $\delta > 0$. The purpose of this work is to study the asymptotic behavior of $\mathbf{v}_g$ by means of an alternative way.

An approximate solution $\mathbf{v}_a = u_a\mathbf{i} + v_a\mathbf{j}$ of Eqs. (1) was found by perturbation methods [3]. In a complementary work presented in this meeting [4], it is shown that $\mathbf{v}_a$ yields an accurate approximation of the inertial trajectories corresponding to the Eqs. (1) in a time interval $[0, t_{\max} \sim 35$ hrs$]$. This time interval is suitable to study the asymptotic character of $\mathbf{v}_g$ by means of $\mathbf{v}_a$. In fact, as expected, the results reported in this work show that $\mathbf{v}_g$ can be obtained by means of a suitable asymptotic expansion of the analytic expression of $\mathbf{v}_a$. Numerical results obtained with analytic expressions of the pressure for tidal oscillations permit us to study the time interval where $\mathbf{v}_g$ is a reliable approximation of Eqs. (1).

References.

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