## Spectral method to compute mesoscale mass-consistent wind fields on a complex terrain

Marco A. Núñez, Jorge Enrique Sánchez-Sánchez, Depto. Física, Universidad Autónoma Metropolitana Iztapalapa, AP 55-534, CP 09340, D.F., México, e-mail: manp@xanum.uam.mx

One of the main physical constraints the that wind field has to satisfy, is the conservation of mass [1,2]. Several methods have been proposed to get mass-consistent wind fields from a data set provided by a meteorological network of numerical solutions of the hydrodynamic equations. These methods go from a simple interpolation to the numerical solution of primitive-equation models in 3D or 4D data-assimilation schemes [3]. Variational Mass consistent models (VMCM's) of the wind field, is a class a class of diagnostic models that is intermediate in sophistication between interpolated and primitive-equation models and attempts to satisfy the continuity equation [4-7]. Several studies give evidence that this models appear to suitable for several applications, since they introduce a fewer number of arbitrary parameters [5-7]. VMCM's have been applied to modeling the transport, diffusion and dispersion of atmospheric pollutants and as input of prognostic models [5-11], a review of these models is given in Refs. [5,6]. The simplicity of VMCM's has motivated the development of new computational algorithms [11-14] and applications for air quality modeling and climatological studies [15,16] over the last decades.

The main aim of this work is to propose a scheme to compute VMCM's. The scheme has the following features: (i) The formulation uses a functional in the space of contravariant vector fields that leads to an elliptic equation which can be solved explicitly by means of trigonometric Fourier series, independently of the complexity of the terrain. This reduces the computational problem to the use of 2D and 3D Fast Fourier Algorithm, for which there are highly efficient computational routines. (ii) The scheme yields an explicit expression of the accuracy of the wind field to satisfy the continuity equation, expression that requires only the estimation of a three-dimensional Fourier Series. (iii) The boundary conditions are easy to use and improve in several orders of magnitude the accuracy with which velocity field satisfies the continuity equation. The method is illustrated by means of analytic and numerical examples to estimate the error produces by the estimation of Fourier coefficients with the Fast Fourier Transform algorithm.

## References

1. Trenberth, K. E., 1991: Climate diagnostics from global analyses: Conservation of mass in ECMWF analyses. J. Climate, 4, 707–722. Trenberth, K. E., J. W. Hurrell, and A. Solomon, 1995: Conservation of mass in three dimensions in global analyses. J. Climate, 8, 692–708.

2. D. W. Byun, Dynamically Consistent Formulations in Meteorological and Air Quality Models for

Multiscale Átmospheric Studies. Part II: Mass Conservation Issues. J. Atmos. Sci. 56, 3808 (1999). N. L. Seaman, "Meteorological modeling for air--quality assessments", Atmos. Environ 34, 2231-2259 (2000).

3. R. Daley, Atmospheric data analysis (Cambridge University Press, New York, 1991).

4. Y. Sasaki, An objective analysis based on the variational method, J Met Soc Jpn 36 (1958), 77-88.

5. C.F. Ratto, R. Festa, C. Romeo, O.A. Frumento and M. Galluzzi, Mass-consistent models for wind fields over complex terrain: The state of the art, Environ. Software 9, 247-268 (1994).

6. G. F. Homicz, Three-Dimensional Wind Field Modeling: A Review, Sandia National Laboratories, report SAN2002-2597 (2002).

7. Guo, X. and Palutikof, J. P. (1990) "A Study of Two Mass-Consistent Models: Problems and Possible Solutions," Boundary-Layer Meteorology, Vol. 53, pp. 303-332. G. Gross, On the applicability of numerical mass-consistent wind field models, Boundary-Layer Meteorology 77, 379-394 (1996).

8. Davis, C. G., Bunker, S. S. and Mutschlecner, J. P. (1984) "Atmospheric Transport Models for Complex Terrain," Journal of Climate and Applied Meteorology, Vol. 23, pp. 235-238.

9. T. Kitada, K. Igarashi, M. Owada, "Numerical analysis of the air pollution in a combined field of land/sea breese and the mountain/valley wind", J. Climite Appl. Met. 25, 767-784 (1986).

10. Ishikawa, H.: 1994, `Mass-Consistent Wind Model as a Meteorological Preprocessor for Tracer Transport Models', J. Appl. Meteorol. 33,733-743.

11. Y. Wang, C. Williamson, D. Garvey, S. Chang, J. Cogan, "Application of a Multigrid Method to a Mass-Consistent Diagnostic Wind Model", J. Appl. Meteorol. 44, 1078 (2005).

12. N. Sanín, G. Montero A finite difference model for air pollution simulation, Advances in Engineering Software 38 358--365 (2007).

D. Carstoiu, V. E. Oltean, G. Gorghiu, A. Olteanu, A. Cernian, Approaches in wind modeling and air quality monitoring systems, Bulletin UASVM, Horticulture 65(2)/2008, pISSN 1843-5254; eISSN 1843-5394 (2008).
C. Flores, H. Juárez, M. A. Núñez, M. L. Sandoval, Algorithms for Vector Field Generation in Mass Consistent Models, Numerical Methods for Partial Differential Equations 26, 826 (2010).

15. S. Finardi, G. Tinarelli, A. Nanni, G. Brusasca, G. Carboni, Evaluation of a 3--D flow and pollutant dispersion modelling system to estimate climatological ground level concentrations in complex coastal sites, Int. J. Environ. Pollution 16, 472--482 (2001).

16. F. Castino, L. Rusca, and G. Solari, Wind climate micro--zoning: a pilot application to Liguria Region (North--Western Italy), J. Wind Eng. Ind. Aerodynam. 91, 1353--1375 (2003).